Straight Line Coordinate Geometry:

Basic Formulae

Midpoint Formulae

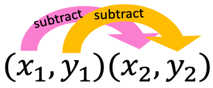
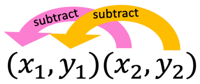
Midpoint between 2 points $(x_1, y_1), (x_2, y_2) \Longrightarrow \text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

In English this formula just says: Add the x coordinates and divide by 2 (i.e. find the average) and add the y coordinates and divide by 2 (i.e. find the average)

Slope/Gradient Formulae

Slope between 2 points $(x_1, y_1), (x_2, y_2) \Longrightarrow \text{slope} = \frac{y_2-y_1}{x_2-x_1}$ OR $\frac{y_1-y_2}{x_1-x_2}$

It doesn't matter which way round we subtract, just so long as we keep the same direction (see the arrows)

Way 1: (left to right)	Way 2: (right to left)
 $m = \frac{y_1 - y_2}{x_1 - x_2}$	 $m = \frac{y_2 - y_1}{x_2 - x_1}$

In English this formula just says: subtract the y coordinates and divide by the answer we get by subtracting the x coordinates. The formula should make sense since $\frac{\text{rise}}{\text{run}} = \frac{\updownarrow}{\leftrightarrow}$ which is $\frac{\text{change in y}}{\text{change in x}}$

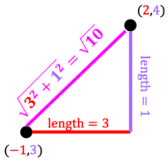
Find the slope between the 2 points $(-1,3)$ and $(2,4)$ Let's colour code as $(-1, 3), (2, 4)$	
Way 1: left to right	Way 2: right to left
$\frac{3-4}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$	$\frac{4-3}{2--1} = \frac{1}{3}$

Distance Formulae

There are 2 ways to find the distance:

Way 1: Build a triangle - We find the x and y distances between the coordinates and use Pythagoras to find the hypotenuse length which is the distance between the points

Way 2 Formula - Distance between 2 points $(x_1, y_1), (x_2, y_2) \Longrightarrow \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the distance between the 2 points $(-1,3)$ and $(2,4)$ Let's colour code as $(-1, 3), (2, 4)$	
Way 1: Build a triangle	Way 2: Formula
 Distance $= \sqrt{3^2 + 1^2} = \sqrt{10}$	Distance $= \sqrt{(2 - -1)^2 + (4 - 3)^2}$ $= \sqrt{3^2 + 1^2} = \sqrt{10}$

How To find Gradients/Intercepts And Link To Equations Of Lines

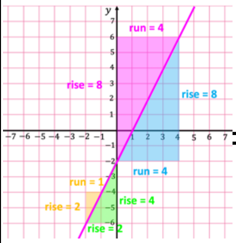
Finding The Gradient/Slope (3 ways)

There are 3 ways to find the gradient dependent on what you're given (graph, points or the equation of a line):

Way 1: If given the graph – Here we have 2 options

- Option 1: pick any 2 points on the graph and use $\frac{\text{rise}}{\text{run}}$ and build a triangle.
  We can build ANY of the triangles shown (it doesn't matter what size they are or whether they are above or below the line). All will give the same answer. Some just need to be simplified in order to see that they give the same value! We can pick any pair of points.

Example 1: Find the slope of following pink line on the graph on the left



ans $\Longrightarrow$

Using the pink triangle : $m = \frac{rise}{run} = \frac{8}{4} = 2$

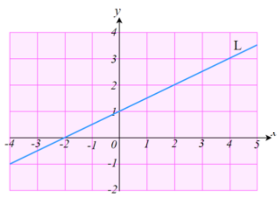
Using the blue triangle. : $m = \frac{rise}{run} = \frac{8}{4} = 2$

Using the orange triangle. : $m = \frac{rise}{run} = \frac{2}{1} = 2$

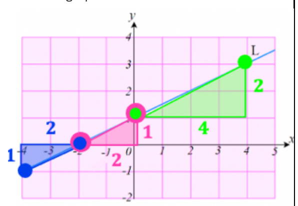
Using the green triangle. : $m = \frac{rise}{run} = \frac{4}{2} = 2$

slope = 2

Example 2: Find the slope of the blue line on the graph on the left



ans $\Longrightarrow$ build triangle

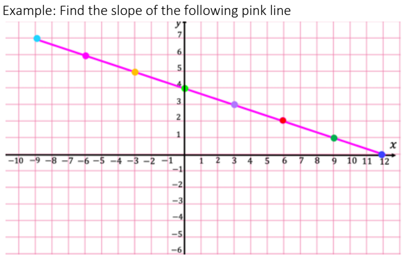


We can use any triangle built to find the slope

slope $= \frac{\text{rise}}{\text{run}} = \frac{1}{2}$ or $\frac{1}{2}$ or $\frac{2}{4} = \frac{1}{2}$

- Option 2: pick any 2 points on the graph and use the slope formula

Example: Find the slope of the following pink line



$\Longrightarrow$

So, for our graph for example 4 on the previous page, we had the coordinates

$(-9,7)$ $(-6,6)$ $(-3,5)$ $(0,4)$ $(3,3)$ $(6,2)$ $(9,1)$ $(12,0)$

Pick ANY pair of coordinates. Let's choose $(-3,5)$ and $(3,3)$

Way 1:	Way 2:
slope $= \frac{5-3}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$	slope $= \frac{3-5}{3--3} = \frac{2}{-6} = -\frac{1}{3}$

Way 2: If given 2 points - Use the slope formula

This is the same as way 1 option 2 above except you're one step ahead since you're given the points and don't have to read them off the graph.

Way 3: If given the equation - We use the letter m to represent the gradient of a line. The equation of a line looks like $y = mx + c$ where m is the gradient and can be read off as the slope.

If we don't have the line in this form we must re-arrange it (see the examples below to help understand this).

$$y = mx + c$$

m: gradient/slope; c: y intercept

$y = 2x - 1$	$y = x + 2$	$y = -x + 4$	$y = -2 + 3x$	$y = 2 - 4x$	$x = 4$	$y = 5$
$y = 2x - 1$ We can spot this straight away gradient = 2	$y = x + 2$ This means the same as $y = 1x + 2$ gradient = 1	$y = x + 2$ This means the same as $y = -1x + 4$	Need to re-order this first since it is not in the form $y = mx + c$ $y = 3x - 2$	Need to re-order this first $y = -4x + 2$ gradient = −4	This is a vertical line since x is the same value the whole time. This has no gradient.	This is a horizontal line since y is the same value the whole time. $y = 5$ is like writing $y = 0x + 5$ gradient = 0

gradient $= -1$ (row)		gradient $= -1$	gradient $= 3$		The gradient here is undefined	
$\mathbf{y+x=4}$	$\mathbf{y-2x=5}$	$\mathbf{2x+4y=5}$	$\mathbf{5x-2y=7}$	$\mathbf{-2x+3y-1=0}$	$\mathbf{x+2y+5=0}$	$\mathbf{y=\frac{x}{3}+2}$
We need to use algebra to re-arrange $y=-x+4$ This means the same as $y=-1x+4$ gradient $= -1$	We need to use algebra to re-arrange $y=2x+5$ gradient $= 2$	We need to use algebra to re-arrange $4y=-2x+5$ $y=\frac{-2x+5}{4}$ We can split this up in order to separate the gradient and y intercept $y=-\frac{2}{4}x+\frac{5}{4}$ We can simplify the fraction $y=-\frac{1}{2}x+\frac{5}{4}$ gradient $= -\frac{1}{2}$	We need to use algebra to re-arrange $-2y=-5x+7$ $y=\frac{-5x+7}{-2}$ We can split this up in order to separate the gradient and y intercept $y=\frac{5}{2}x-\frac{7}{2}$ gradient $= \frac{5}{2}$	We need to use algebra to re-arrange $3y=2x+1$ $y=\frac{2x+1}{3}$ We can split this up in order to separate the gradient and y intercept $y=\frac{2}{3}x+\frac{1}{3}$ gradient $= \frac{2}{3}$	We need to use algebra to re-arrange $2y=-x-5$ $y=\frac{-x-5}{2}$ We can split this up in order to separate the gradient and y intercept $y=-\frac{1}{2}x-\frac{5}{2}$ gradient $= -\frac{1}{2}$	means $y=\frac{1}{3}x+2$ gradient $= \frac{1}{3}$

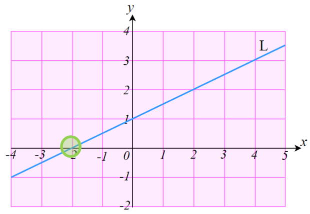
Note: If you're given the equation of another line parallel/perpendicular to you can also spot the gradient.

- Told parallel to another line⟹ locate m for this line (using way 3) and use same slope
- Told perpendicular to another line⟹ locate m for this line (using way 3) and don't use the same slope but "flip the fraction and change the sign" (this is just the fact that perpendicular slopes multiply to make -1. Let's look at some examples for perpendicular lines as this is a hard concept for some students.

  - If a line has slope 2, what slope would a perpendicular line have?
    slope 2 means the same thing as $\frac{2}{1}$. Flipping the fraction gives $\frac{1}{2}$. Changing the sign means we have a negative, so $-\frac{1}{2}$. Hence a perpendicular line has slope $-\frac{1}{2}$.
    Let's check if we have done this correctly by checking if the slopes multiply to make -1:
    $2\left(-\frac{1}{2}\right)=-1$. Yes, they do, as we expected!
  - If a line has slope $-\frac{2}{3}$, what slope would a perpendicular line have?
    Flipping the fraction gives $\frac{3}{2}$. Changing the sign means we have a positive. Hence a perpendicular line has slope $\frac{3}{2}$.
    Let's check if we have done this correctly by checking if the slopes multiply to make -1:
    $-\frac{2}{3}\left(\frac{3}{2}\right)=-1$. Yes, correct again!
  - If a line has slope $\frac{1}{3}$, what slope would a perpendicular line have?
    Flipping the fraction gives $\frac{3}{1}$. Changing the sign means we have a negative so $-\frac{3}{1}$. Hence a perpendicular line has slope $-\frac{3}{1}$ which is just -3.
    Let's check if we have done this correctly by checking if the slopes multiply to make -1:
    $\frac{1}{3}(-3)=-1$. Yes, correct again!

Finding the x intercept (2 ways)

Way 1: If given the graph
x intercept is just the x value where the line crosses the x axis



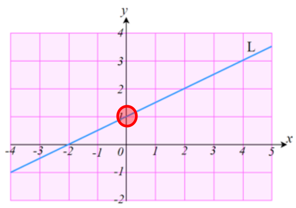
$x=-2$
$(-2,0)$

Way 2: If given the equation
set $y=0$ (we replace y with 0) and solve for x. The coordinate will be $(x,0)$ where x is the value found

Find the x intercept of the line $2y-x=2$
We replace y with 0
$2(0)-x=4$
$0-x=2$
$-x=2$
$x=-2$
$(-2,0)$

Finding the y intercept (2 ways)

Way 1: If given a graph
y intercept is just where the line crosses the y axis.



$y=1$
$(0,1)$

Way 2: If given the equation

- Option 1: set $x=0$ (replace x with 0) and solve for y. The coordinate will be $(0,y)$ where y is the value found.

  Find the y intercept of the line $2y-x=2$
  We replace x with 0
  $2y-0=2$
  $2y=2$
  $y=1$
  $(0,1)$

- Option 2: We use the letter c to represent the y intercept on a line. We get into the line equation form $y=mx+c$ and locate c. Let's see the examples below.

$$\boldsymbol{y = mx + c}$$

gradient/slope (m) y intercept (c)

$\mathbf{y=x-2}$	$\mathbf{y=2x-1}$	$\mathbf{y=-x+4}$	$\mathbf{y=-2+3x}$	$\mathbf{y=2-4x}$	$\mathbf{x=4}$	$\mathbf{y=5}$
$y=x-2$ y intercept is -2 $(0,-2)$	$y=2x-1$ y intercept is -1 $(0,-1)$	$y=-x+4$ y intercept is 4	Need to re-order this first $y=3x-2$ y intercept is -2 $(0,-2)$	Need to re-order this first $y=-4x+2$ y intercept is 2 $(0,2)$	This is a vertical line since x is the same value the whole time. There is no y intercept	This is a horizontal line since y is the same value the whole time. $y=5$ is like writing $y=0x+5$

		(0,4)				y intercept is 5 (0,5)
$y+x=4$	$y-2x=5$	$2x+4y=5$	$5x-2y=7$	$2x+3y-1=0$	$x+2y+5=0$	
We need to use algebra to re-arrange $y=-x+4$ y intercept is 4 (0,4)	We need to use algebra to re-arrange $y=2x+5$ y intercept is 5 (0,5)	We need to use algebra to re-arrange $4y=-2x+5$ $y=\frac{-2x+5}{4}$ $y=-\frac{1}{2}+\frac{5}{4}$ y intercept is $\left(0,\frac{5}{4}\right)$	We need to use algebra to re-arrange $-2y=-5x+7$ $y=\frac{-5x+7}{-2}$ $y=\frac{5}{2}x-\frac{7}{2}$ y intercept is $\left(0,-\frac{7}{2}\right)$	We need to use algebra to re-arrange $3y=-2x+1$ $y=\frac{-2x+1}{3}$ $y=-\frac{2}{3}x+\frac{1}{3}$ y intercept is $\left(0,\frac{1}{3}\right)$	We need to use algebra to re-arrange $2y=-x-5$ $y=\frac{-x-5}{2}$ $y=-\frac{1}{2}x-\frac{5}{2}$ y intercept is $\left(0,-\frac{5}{2}\right)$	

Find The Equation Of A Line

Ultimately you need just 2 things in order to find this – the gradient and a point (x, y)

Way 1: If using $y = mx + c$ form (most common form used)

We use the letter m to represent slope and c to represent y intercept.

$$y = mx + c$$

If we can find the gradient m and y intercept c then we are done.

Step 1: Find the gradient m using one of the 3 ways in the finding gradient row

Step 2: Find the y intercept c by either reading it off the graph (if given the graph) or plugging the point given (x, y) into equation with slope and solving/re arranging for c using algebra. Make sure you plug in the correct point that the line passes through!

Way 2: If using $y - y_1 = m(x - x_1)$ form (this is for the A level coure)

We use the letter m to represent slope and (x, y) as the point the line passes through.

$$y - y_1 = m(x - x_1)$$

If we can find the gradient m and plug in the point (x, y) then we are done.

Step 1: Find the gradient m using one of the ways in the finding gradient row above

Step 2: plug the point (x, y) into x_1 and y_1

Note: There are 3 forms that you can be asked to leave your answer in:

1)Slope intercept Form: $y = mx + c$

Way 1 gives you this or way 2 but with way 2 you'll need to multiply out the brackets and move terms on the other side after to get y on its own on the left-hand side)

2) Point Slope Form: $y - y_1 = m(x - x_1)$

Way 2 gives you this straight away

3) General Form: $ax + by = c$

We are not allowed fractions in this form. To get this form we put all the terms from form 1 on one side and multiply all terms by the denominators to get rid of the fractions (if we have them)

Level 1 Example:
Find the equation of the line which is parallel to $y = 3x - 8$

Level 2 Example:
Find the equation of the line which is parallel to $3y + 6x = 7$and passes through the point (1,3)

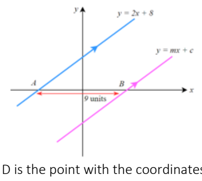
Level 3 example:
What is the equation of the line that is perpendicular to the line $y = 4x - 9$, which passes through the point (0.−3)

Level 4 example:
Find the equation of the line that passes through the points $(2, -4)$ and (5,10)

Level 5 example:
There are 3 points A(1,-5), B(2,−1) and C(0,5). The line L is parallel to AB and passes through C. Find the equation of the line.

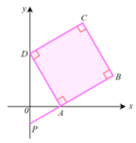
Level 6 example:
P has coordinates $(-9, 7)$. Q has coordinates $(11,12)$. M is the midpoint of the line segment PQ. Line L is perpendicular to the line segment PQ. L passes through M. Find an equation for L

Level 7 example:
The line $y = mx + c$ is parallel to the line $y = 2x + 8$. Find the value of m and the value of c



Level 8 example:
ABCD is a kite with AB=AD and CB=CD. B is the point with the coordinates (10,19). D is the point with the coordinates (2,7). Find the equation of the line AC in the form $px + qy = r$, where p, q and r are integers

Level 9 example:
ABCD is a square. P and D are points on the y axis. A is a point on the x axis. PAB is a straight line. The equation of the line that passes through the points A and D is $y = -2x + 6$. Find the length of PD.



Answers

$y = 3x +$ any value you want, $y = -2x + 5$, $y = -\frac{1}{4}x - \frac{11}{4}$, $y = 4x + 5$ $y = \frac{3}{4}x + \frac{7}{2}$, , $y = -4x + \frac{27}{2}$,
$m = 2$ and $c = -10$, $2x + 3y = 51$, distance=7.5

Other Types Of Questions

Intersection Points Of Two Lines

This finds where the 2 lines cross, so we just solve simultaneously as this is what solving simultaneously finds us (using our knowledge of ow how simultaneous equations which is not covered in this sheet). Recall that we can use elimination or substitution to solve simultaneous equations. Our solution is the intersection point.

Example
Find the point where the graphs of the lines $6x + 2y = -3$ and $4x - 3y = 11$ intersect

Solving simultaneously gives $x = 0.5, y = -3$ so we have the intersection point $(0.5, -3)$

Showing A Point Lies On A Line

3 possible methods

- Way 1: Plug in x and solve for y and show this is the y you need
- Way 2: Plug in y and solve for x and show this is the x you need
- Way 3: Plug in x and y and show you get a true result

Example 1:
Show that the point $(1,5)$ lies on the line $x + 2y = 11$

let $x = 1$ and check if we get $y = 5$

$1 + 2y = 11$
$2y = 10$
$y = 5$ which is correct

Finding Missing Coordinates In Triangles and Quadrilaterals

Right-Angled Triangles (2 possible methods)

- Way 1: Use fact that perpendicular i.e. slopes multiply to make -1 and form an equation based on this. Use algebra to solve for the unknown
- Way 2: Use Pythagoras with the distance formula to form an equation and use algebra to solve for unknown

Parallelograms (2 possible methods)

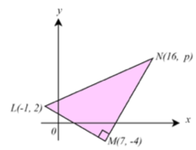
- Way 1: Use fact that opposite sides are the same length (use distance formula)
- Way 2: Use fact that how you get from one coordinate to another should always be the same (i.e. work out what you do to x and y and apply this from another coordinate in order to find the missing coordinate).

Example 1:
The figure above shows a right-angled triangle LMN. The points L and M have coordinates (–1, 2) and (7, –4) respectively.

i. Find an equation for the straight line passing through the points L and M. Give your answer in the form $ax + by + c = 0$, where a, b and c are integer

Given that the coordinates of point N are (16, p), where p is a constant, and angle LMN = 90°

ii. find the value of p
iii. Given that there is a point K such that the points L, M, N, and K form a rectangle, find the y coordinate of K.



(ans $3x + 4y - 5 = 0, p = 8$, y coordinate is 14)

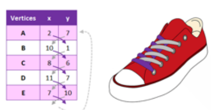
Finding the area Area Of ANY Shape (if given the coorindates)

We can either use

- Our usual formula area of shapes
  - Area of triangle $= \frac{1}{2} \times b \times h$.
    Note: Might need to draw the height in need to locate it
    Remember that the height doesn't bisect a side unless we have isosceles or equilateral triangle
  - Quadrilateral: Break into 2 triangles

Or we can use a really cool formula that finds us the area of ANY shape (as long as weare given the coordinates)

- Shoelace Formula/Gauss' Area Formula

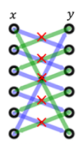


Step 1: Plots the coordinates

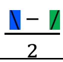
Step 2: Start at ANY coordinate

Step 3: Go anti-clockwise around the shape and write down all vertices as a vertical list. Make sure you "close the shape" at the end by re-writing the first coordinate you started with.

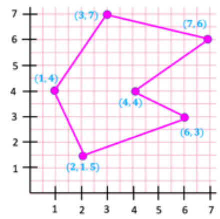
Step 4: Cross multiply corresponding diagonal coordinates and add.
(First going from left to right and adding all results together and then going from right to left and adding all results together)



Step 5: Subtract these two answers and then divide by 2



Example 1:
Find the area of the following shape



Let's pick a point to start at: $(2,1.5)$ and write the coordinates going anti-clockwise

(2,1.5)
(6,3)
(4,4)
(7,6)
(3,7)
(1,4)
(2,1.5)

Let's colour code to explain the method.

Left bracket	Right bracket
(2,1.5)	(2,1.5)
(6,3)	(6,3)
(4,4)	(4,4)
(7,6)	(7,6)
(3,7)	(3,7)
(1,4)	(1,4)
(2,1.5)	(2,1.5)

$$\frac{((2\times3)+(6\times4)+(4\times6)+(7\times7)+(3\times4)+(1\times1.5)) - ((1.5\times6)+(3\times4)+(4\times7)+(6\times3)+(7\times1)+(4\times2))}{2}$$

=17.25

For harder questions you must know your geometry

- Perpendicular bisectors cut a line in half
- Diagonals of a rhombus, kite square are perpendicular (not true for a rectangle though)
- Only one diagonal of a kite is bisected whereas both diagonals of a rhombus and square are bisected
- Angles of a parallelogram and rectangle are not bisected by the diagonals